BMEN 3010 — HOMEWORK 2 SOLUTIONS

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[1] (2 POINTS) CALCULATE F_m, F_J, α FOR β AS GIVEN

FROM COURSE NOTES:

$$F_M = \frac{\left(\frac{Wc}{2} - W_t b\right)\cos\beta}{a \sin\theta}$$

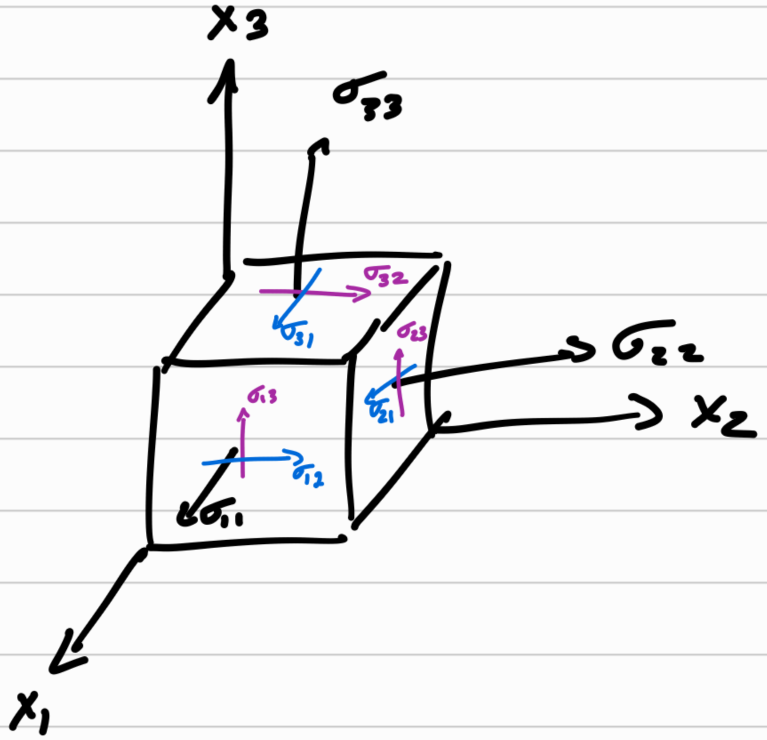
$$F_J = \sqrt{F_m^2 + \left(\frac{W}{2} - W_t\right)^2 + 2F_m\left(\frac{W}{2} - W_t\right)\sin(\beta-\theta)}$$

$$\alpha = \tan^{-1}\left[\frac{\frac{W}{2} + F_m \sin(\beta-\theta) - W_t}{F_m \cos(\beta-\theta)}\right]$$

β	F_m	α	F_J
90	0	90°	0.44W
45	4.49W	34.6°	4.73W
36	5.5W	19.3°	5.63W

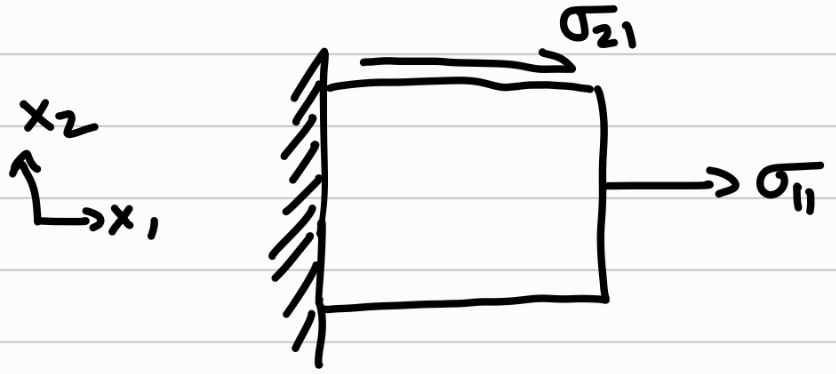
∴ ① EVEN THOUGH EACH LEG SUPPORTS $\frac{W}{2}$ BODY WEIGHT, MUSCLE + JOINT FORCES CAN EXCEED BODY WEIGHT BY 5X.

2



3

STRESS IS A 2ND ORDER TENSOR B/C IT IS ASSOCIATED WITH 2 DIRECTIONS: ONE REPRESENTING THE DIRECTION OF THE ASSOCIATED FORCE, AND THE OTHER FOR THE ORIENTATION OF THE SURFACE IT ACTS ON. IN THIS WAY, WE CAN DISTINGUISH BETWEEN SHEARING (σ_{21} below) AND TENSION (σ_{11}), EVEN THOUGH BOTH INVOLVE A FORCE ACTING IN THE X_1 DIRECTION



EACH WOULD PRODUCE DISTINCT DEFORMATIONS SO WE MUST HAVE A WAY TO DISTINGUISH THEM $\Rightarrow$ 2ND ORDER TENSOR.

$\boxed{4}$

$$u_1 = 0.05x_1 + 0.025x_2 + 0.05$$

$$u_2 = x_1 - 0.02x_2 + 0.05$$

$$\underset{\sim}{\varepsilon} = \frac{1}{2}\left[\nabla \otimes \underset{\sim}{u} + \nabla \otimes \underset{\sim}{u}^T\right]$$

$$\varepsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]$$

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = 0.05$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = -0.02$$

$$\varepsilon_{12} = \frac{1}{2}\left[\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right]$$

$$= \frac{1}{2}\left[0.025 + 1\right] = \frac{1.025}{2} = 0.5125$$

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} 0.05 & 0.5125 & 0 \\ 0.5125 & -0.02 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\boxed{5}$

- FIND UNIT VECTORS

$$\underset{\sim}{e}_1' = \frac{1}{3}(2\underset{\sim}{e}_1 + 2\underset{\sim}{e}_2 + \underset{\sim}{e}_3)$$

$$\underset{\sim}{e}_2' = \frac{1}{3\sqrt{5}}(3\underset{\sim}{e}_1 - 6\underset{\sim}{e}_3) = \frac{1}{\sqrt{5}}(\underset{\sim}{e}_1 - 2\underset{\sim}{e}_3)$$

$$\underset{\sim}{e}_1' \underset{\approx}{\epsilon} \cdot \underset{\sim}{e}_2' = \epsilon_{ij}\, e_1' \cdot \underset{\sim}{e}_i \otimes \underset{\sim}{e}_j \cdot \underset{\sim}{e}_2'$$

$$= \epsilon_{ij} \frac{1}{3}(2\underset{\sim}{e}_1 + 2\underset{\sim}{e}_2 + \underset{\sim}{e}_3) \cdot \underset{\sim}{e}_i \otimes \underset{\sim}{e}_j \cdot \frac{1}{\sqrt{5}}(\underset{\sim}{e}_1 - 2\underset{\sim}{e}_3)$$

$$= \frac{\epsilon_{ij}}{3\sqrt{5}}(2\delta_{1i} + 2\delta_{2i} + \delta_{3i})(\delta_{1j} - 2\delta_{j3})$$

$$= \frac{\epsilon_{ij}}{3\sqrt{5}}\big(2\delta_{1i}\delta_{1j} + 2\delta_{2i}\delta_{1j} + \delta_{3i}\delta_{1j}$$

$$- 4\delta_{1i}\delta_{j3} - 4\delta_{2i}\delta_{j3} - 2\delta_{3i}\delta_{j3}\big)$$

also accept matrix multip. $[e_1'][\epsilon][e_2']$

$$= \frac{1}{3\sqrt{5}}\big(2\epsilon_{11} + 2\epsilon_{21} + \epsilon_{31} - 4\epsilon_{13} - 4\epsilon_{23} - 2\epsilon_{33}\big)$$

$$= \frac{10^{-3}}{3\sqrt{5}}(10 + 6 + 0 - 0 + 4 - 4)$$

$$= \frac{16 \times 10^{-3}}{3\sqrt{5}}$$

FROM $\underline{4.5}$,

$$\epsilon_{12} = \frac{16 \times 10^{-3}}{3\sqrt{5}} = \frac{1}{2}\Big(\overbrace{\frac{\pi}{2} - \theta_{12}'}^{\text{change in angle}}\Big)$$

$$\Rightarrow \quad 2\epsilon_{12} = \Delta\theta = \frac{32 \times 10^{-3}}{3\sqrt{5}} = \boxed{4.77 \times 10^{-3}}$$

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A CONSTRAINED GROWTH WHERE EXPANSION (e.g. THROUGH CELL PROLIFERATION OR CELL HYPERTROPHY) LEADS TO EXTENSIONAL STRAIN BUT AS THE TISSUE GROWS, IT IS INCREASINGLY COMPRESSED BY NEIGHBORING TISSUE. TUMOR GROWTH IS AN EXAMPLE.

B IF A TISSUE SHORTENS AGAINST A CONSTRAINT, SUCH AS CONTRACTION OF SKELETAL MUSCLE, TENSION INCREASES, INDICATING POSITIVE STRESS, BUT TISSUE SHORTENING ⇒ NEGATIVE STRAIN.